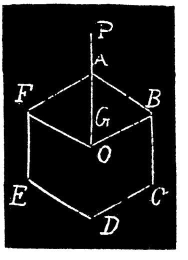
MECHANICS.

153. Proposed by W. J. GREENSTREET, A. M., Editor of The Mathematical Gazette, Stroud, Eng.

An equiangular polygon consisting of equal, freely jointed rods, is hung up from vertex, A. The vertices adjacent to A are connected by a light rod of such length that the polygon is still regular. Find the stress in the rod and the reactions at the vertices.

Solution by G. B. M. ZERR, A. M., Ph. D., Professor of Chemistry and Physics, The Temple College, Philadelphia, Pa.

Let P be the point of suspension, O, the centroid of the system, T, the stress on the weightless rod FB, nW, the weight of the rods, $FB=x$, $AB=a$, $AO=y$, $\angle OAB=\angle OBA=\theta$, R=reaction.



Suppose the deformation such that the centroid moves vertically through a small space. Then $Tdx-nWdy=0$.

$BG : AB=\sin\theta : 1$; $\therefore$ $x=2a\sin\theta$.

$AO : AB=\sin\theta : \sin(\pi-2\theta)$; $\therefore$ $y=a\sin\theta/\sin(\pi-2\theta)$.

$\therefore$ $y=a\sin\theta/\sin2\theta=\frac{1}{2}a\sec\theta$.

$\therefore$ $dx=2a\cos\theta d\theta$, $dy=\frac{1}{2}a\sec\theta\tan\theta d\theta$.

$\therefore$ $2aT\cos\theta d\theta=\frac{1}{2}(nWa)\sec\theta\tan\theta d\theta$.

$\therefore$ $T=\frac{1}{4}nW\sec^2\theta\tan\theta$. R acts along BO, $\angle ABG=\frac{1}{2}\pi-\theta$.

$\therefore$ $\angle OBG=\angle OFG=2\theta-\frac{1}{2}\pi$, $\angle BOF=2\pi-4\theta$.

$\therefore$ $R : T=\sin(2\theta-\frac{1}{2}\pi) : \sin(2\pi-4\theta)$; $R : T=\cos2\theta : \sin4\theta$.

$\therefore$ $R=T\cos2\theta/\sin4\theta=T/2\sin2\theta=T/4\sin\theta\cos\theta$. $\therefore$ $R=\frac{1}{16}nW\sec^4\theta$.

Also solved by *G. W. GREENWOOD.*

154. Proposed by M. E. GRABER, Graduate Student, Heidelberg University, Tiffin, Ohio.

Find the form of the curve in a vertical plane, such that a heavy bar resting on its concave side and on a peg at a given point, (the origin), may be at rest at all positions.

Solution by G. W. GREENWOOD, A. B., Professor of Mathematics, McKendree College, Lebanon, Ill.; and E. L. SHERWOOD, Professor of Mathematics, Shady Side Academy, Pittsburg, Pa.

Let the bar be homogeneous and of length $2b$. Assume the peg and the curve to be smooth. Take the peg as origin, the horizontal line through it in the plane as initial line, and measure θ downwards.

The coördinates of the lower extremity being (r, θ), those of the center of the rod are $(r-b, \theta)$. Since in all positions the rod is in equilibrium, the center is at a constant distance from the initial line. Hence $(r-b)\sin\theta=a$, is the curve traced by one extremity.

Also solved by *G. B. M. ZERR.*

DIOPHANTINE ANALYSIS.

107. Proposed by L. C. WALKER, A. M., Graduate Student, Leland Stanford Jr. University, Cal.

Required the least three positive integral numbers such that the sum of all three of them and the sum of every two of them shall be a square number.